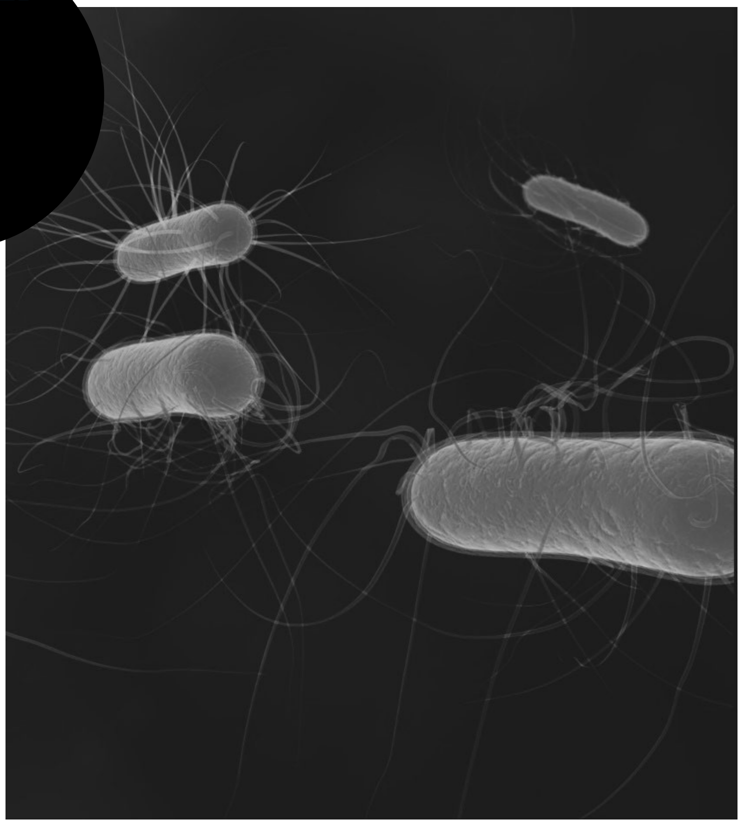
CHAPTER 2

Logarithm



For updates

[SLO: M-09-A-05]: [Summative-U]	Express a number in scientific notations and vice versa	

Knowledge 2.2: Logarithm

[SLO: M-09-A-06]: [Formative-K]	Describe Logarithm of a Number	

Knowledge 2.3: Natural Logarithm

Knowledge 2.4: Anti-logarithm

Knowledge 2.5: Laws and Properties of Logarithm,

[SLO: M-09-A-06]: [Summative-K]	Differentiate b/w Common and Natural algorithm	

Knowledge 2.6: Real world Application of Logarithm

[SLO: M-09-A-08]: [Summative-A]	Apply laws of logarithm to real life situations such as growth and decay, loudness of sound.	

[SLO: M-09-A-05]: [Summative-U]	Express a number in scientific notations and vice versa

Q1. Mark the right option. Each part carries one mark.

1. What is the range for the coefficient in a correctly written number in standard form?

A) Between 0 and 1
B) Between 1 and 10
C) Between 10 and 100
D) Any positive number

2. If the diameter of a bacterial cell is 4.2×10^{-6} meters, what is its diameter in ordinary form?

A) 0.0000042
B) 0.000042
C) 0.00000042
D) 4.2×10^{6}

3. Which of the following numbers is correctly written in scientific notation?

A) 3.45×10^{3}
B) 45.6×10^{-2}
C) 0.67×10^{5}
D) 7.89×10^{0}

4. Which number is equivalent to 5.67×10^{8}

A) 567,000,000
B) 56,700,000
C) 5,670,000
D) 5,670,000,000

5. Two distances are given: 7.2×10^{4} meters and 3.6×10^{5} meters. Which distance is shorter?

A) 7.2×10^{4}
B) 3.6×10^{5}
C) Both are equal
D) Cannot be determined

6. Arrange the following numbers in ascending order:

A) $2.3\times10^{-3}, 5.4\times10^{-4}, 1.1\times10^{-2}, 7.8\times10^{-5}$
B) $5.4\times10^{-4}, 7.8\times10^{-5}, 2.3\times10^{-3}, 1.1\times10^{-2}$
C) $7.8\times10^{-5}, 5.4\times10^{-4}, 2.3\times10^{-3}, 1.1\times10^{-2}$
D) $1.1\times10^{-2}, 2.3\times10^{-3}, 5.4\times10^{-4}, 7.8\times10^{-5}$

7. Given the two numbers 5.6×10^{7} and 4.3×10^{5}, which of the following is their sum expressed in scientific notation?

A) 5.64×10^{7}
B) 5.6×10^{8}
C) 6.0×10^{7}
D) 5.6043×10^{7}

8. Convert 6.7×10^{4} and 2.5×10^{-3} to standard form and then calculate their product.

A) 167.5×10^{1}
B) 1.675×10^{2}
C) 1.675×10^{3}
D) 1675×10^{1}

9. Which of the following numbers is greater?

A) 4.5×10^{3}
B) 3.2×10^{4}

C) 6.8×10^2

D) 7.9×10^1

10. What is the result when dividing 8.4×10^6 with 2.1×10^2 using scientific notation rules?

A) 4×10^4

C) 40×10^2

B) 4×10^8

D) 4×10^3

Short Response Questions

1. Simplify the answer in Standard form $\frac{(2.5\times10^6)\times(3.0\times10^{-2})}{(5.0\times10^4)}-(7.5\times10^2)$

Solution:

To simplify the given expression in standard form:

$$\frac{(2.5\times10^6)\times(3.0\times10^{-2})}{(5.0\times10^4)}-(7.5\times10^2)$$

Multiply the terms in the numerator:

$$(2.5\times10^6)\times(3.0\times10^{-2})=7.5\times10^{6+(-2)}=7.5\times10^4$$

Divide the result by the denominator:

$$\frac{7.5\times10^4}{5.0\times10^4}=1.5$$

Subtract the second term:

$$1.5-(7.5\times10^2)=1.5-750$$

$$1.5-750=-748.5$$

Now, we will express in standard form:

$$-748.5=-7.485\times10^2$$

2. Simplify the following expression and express your answer in scientific notation:

$(3.6\times10^5)\times(4.2\times10^{-3})+\frac{5.4\times10^2}{1.8\times10^{-1}}$

Solution:

Let's simplify the given expression in scientific notation:

$$(3.6\times10^5)\times(4.2\times10^{-3})+\frac{5.4\times10^2}{1.8\times10^{-1}}$$

Multiply the first part:

$$(3.6\times10^5)\times(4.2\times10^{-3})=15.12\times10^{5+(-3)}=15.12\times10^2$$

Now, express in scientific notation:

$$15.12\times10^2=1.512\times10^3$$

Divide the second part:

$$\frac{5.4\times10^{2}}{1.8\times10^{-1}}=\frac{5.4}{1.8}\times10^{2-(-1)}=3\times10^{3}$$

Add the results and we get the answer in scientific notation is:

$$1.512\times10^{3}+3\times10^{3}=(1.512+3)\times10^{3}=4.512\times10^{3}$$

3. A factory produces 2.5×10^{6} units of a product each year. If each unit requires 1.2×10^{-4} kilograms of raw material, calculate the total amount of raw material required for one year in kilograms. Then, if the raw material costs 2.0×10^{2} dollars per kilogram, find the total cost for the raw material in scientific notation.

Solution:

The factory produces 2.5×10^{6} units of a product each year, and each unit requires 1.2×10^{-4} kilograms of raw material.

To find the total raw material required, multiply the number of units by the raw material required per unit:

$$\text{Total raw material}=(2.5\times10^{6})\times(1.2\times10^{-4})$$
$$=3.0\times10^{6+(-4)}=3.0\times10^{2}\text{ kilograms}$$

The raw material costs 2.0×10^{2} dollars per kilogram. To find the total cost, multiply the total amount of raw material by the cost per kilogram:

$$\text{Total cost}=(3.0\times10^{2})\times(2.0\times10^{2})$$
$$=6.0\times10^{2+2}=6.0\times10^{4}\text{ dollars}$$

The total cost for the raw material in scientific notation is:

$$6.0\times10^{4}\text{ dollars}$$

4. The radius of Earth is approximately 6.37×10^{6} meters, and the radius of a proton is approximately 1.0×10^{-15} meters. Calculate how many times the radius of Earth is greater than the radius of a proton. Express your answer in scientific notation and explain each step of your calculation.

Solution:

Let's calculate how many times the radius of Earth is greater than the radius of a proton by dividing the radius of earth by the radius of a proton.

Given that the radius of earth is 6.37×10^{6} meters and radius of a proton is 1.0×10^{-15} meters.

The ratio of the radius of earth to the radius of a proton is:

$$\frac{\text{Radius of Earth}}{\text{Radius of proton}}=\frac{6.37\times10^{6}}{1.0\times10^{-15}}$$

$$=6.37\times10^{6-(-15)}=6.37\times10^{6+15}=6.37\times10^{21}$$

The radius of earth is 6.37×10^{21} times greater than the radius of a proton.

[SLO: M-09-A-06]: [Formative-K]	Describe Logarithm of a Number

[SLO: M-09-A-07]: [Summative-K]	Differentiate b/w Common and Natural algorithm

Q1. Mark the right option. Each part carries one mark.

1. What is the base of a common logarithm?

A) e
C) 2
B) 10
D) 5

2. Which logarithm is represented by ln(x)?

A) Common logarithm
C) Binary logarithm
B) Natural logarithm
D) Logarithm base 5

3. Why are natural logarithms particularly useful in mathematics and science?

A) They have a base of 10, which is common in scientific notation.
B) They use the base e, which naturally occurs in exponential growth and decay.
C) They simplify complex calculations involving logarithms.
D) They have no specific use; it's just a convention.

4. If log10(100) = x, what is the value of *x*?

A) 1
C) 3
B) 2
D) 4

5. Solve for x: ln(x) = 3.

A) $x = e^3$
C) $x = 9$
B) $x = 3^e$
D) $x = 1$

6. Find the value of x if $\log_{10}(x) = 4$.

A) 100
C) 10,000
B) 1000
D) 1,000,000

7. If $e^x = 20$, what is $\ln(20) = x$?

A) 2.3
C) ln20
B) 3
D) 4

8. Calculate the value of x if ln(x) = 0.

A) 0
C) e
B) 1
D) 10

9. Determine x if $2\log_{10}(x) = 6$.

A) 100
C) 1000
B) 500
D) 10

10. Given the equation $e^{2x} = 50$, solve for x using logarithms.

A) $\ln(50)/2$
B) ln (25)

C) 2ln(50) D) x = 50

11. If you were to choose a logarithm to model population growth, which one would be more appropriate and why?

A) Common logarithm, because it's easier to calculate.

B) Natural logarithm, because it deals with continuous growth.

C) Either, as both can model growth equally well.

D) Neither, as logarithms are not used in growth modeling.

Short Response Questions:

1- Find the unknown $ln(3x+2)=4$

Solution:

To solve for x in $\ln(3x+2)=4$

If $\ln(y)=z$, then $y=e^z$. Apply this to both sides of the equation:

$$3x+2=e^4,\ 3x=e^4-2$$

Divide both sides by 3: $x=\frac{e^4-2}{3}$. Since, $e^4\approx 54.598$

$$x\approx\frac{54.598-2}{3}=\frac{52.598}{3}\approx 17.53$$

Thus, $x\approx 17.53$

2- A bacteria culture grows according to the formula $N(t)=N_0e^{kt}$. If the initial population is 1000 and doubles in 3 hours, find the growth rate k.

Solution:

The bacteria culture grows according to the formula:

$$N(t)=N_0e^{kt}$$

where: $N(t)$ is the population at time t, N_0 is the initial population, k is the growth rate, and t is time.

We are told that the initial population $N_0=1000$ and that the population doubles in 3 hours. So, $N(t)=2N_0$ when $t=3$. Since the population doubles, we know:

$$2N_0=N_0e^{3k}$$

Substitute $N_0=1000$:

$$2=e^{3k}$$

Take the natural logarithm "*In*" of both sides to eliminate the exponential:

$\ln(2)=3k \implies k=\frac{\ln(2)}{3}$. Since $\ln(2)\approx 0.693$, then $k\approx\frac{0.693}{3}\approx 0.231$

The growth rate k is approximately 0.231 per hour.

[SLO: M-09-A-08]: [Summative-A]	Apply laws of logarithm to real life situations such as growth and decay, loudness of sound.

Q1. Mark the right option. Each part carries one mark.

1. Which of the following represents the product rule of logarithms?

A) $\log_b(x+y)=\log_b(x)+\log_b(y)$
B) $\log_b(x/y)=\log_b(x)-\log_b(y)$
C) $\boldsymbol{\log_b(xy)=\log_b(x)+\log_b(y)}$
D) $\log_b(x^y)=y\log_b(x)$

2. If $\log_3(9)=x$, what is the value of x using the laws of logarithms?

A) 1
B) 3
C) 2
D) 4

3. Given that $\log_b(x)=2$ and $\log_b(y)=3$, find the value of $\log_b(xy^2)$.

A) 8
B) 7
C) 6
D) 12

4. Simplify the expression $\log(100)+\log(0.01)$ using the laws of logarithms.

A) 0
C) -1
B) 1
D) -2

5. If $\log_2(x)+\log_2(4)=5$, what is the value of x ?

A) 2
B) 16
C) 8
D) 32

6. Given that $\log(2)=0.3010$ and $\log(5)=0.6990$, calculate $\log(10)$ using the properties of logarithms.

A) 0.3010
B) 0.9990
C) 1.0000
D) 1.3010

7. If $\log_b(2)=a$ and $\log_b(3)=c$, express $\log_b(18)$ in terms of a and c.

A) $\boldsymbol{a+2c}$
B) $2a+c$
C) $a+c+1$
D) $a+c+\log_b(6)$

8. Use the change of base formula to simplify $\log_5(125)$.

A) $\dfrac{\log(125)}{\log(5)}$
B) 3
C) 2
D) $\dfrac{1}{2}$

9. Given that $\log_4(x)+\log_4(y)=3$ and $\log_4(x/y)=1$, find the values of x and y.

A) $x=2, y=8$
B) $x=4, y=2$
C) $x=8, y=2$
D) $\boldsymbol{x=16, y=4}$

10. A bacterial culture grows according to the formula $N(t)=N_0e^{kt}$. If the initial population is 200 and doubles after 3 hours, what is the value of ?

A) $\frac{\ln(2)}{3}$

B) $\ln(2)$

C) $\frac{\ln(3)}{2}$

D) $\frac{\ln(3)}{4}$

11. If the half-life of a radioactive substance is 5 years, and the amount remaining after t years is given by $A=A_0\cdot e^{kt}$, which logarithm expression would you use to solve for t ?

A) $\log\left(\frac{A}{A_0}\right)$

B) $\ln\left(\frac{A}{A_0}\right)=kt$

C) $\frac{\ln(2)}{5}$

D) $\log\left(\frac{A_0}{A}\right)=kt$

12. The formula for calculating the intensity of an earthquake on the Richter scale is $= \log\left(\frac{I}{I_0}\right)$. If an earthquake has an intensity of $I=10^3 I_0$. what is its magnitude on the Richter scale?

A) 1

B) 3

C) 2

D) 4

13. The brightness of a star is calculated using $B=2.5\log(I/I_0)$. If $I=100I_0$, what is the brightness B ?

A) 5

B) 10

C) 2.5

D) 7.5

Extensive Response Question

1. The loudness of sound in decibels is given by the formula $L=10\log\left(\frac{I}{I_0}\right)$, where I is the intensity of the sound and I_0 is the reference intensity. A sound has an intensity of $I=5\times 10^{-6}$ watts per square meter, and the reference intensity $I_0=10^{-12}$ watts per square meter.

a) Calculate the loudness of the sound in decibels.

b) If the intensity is doubled, what will be the new loudness in decibels?

Solution:

(a): Calculating the loudness of the sound in decibels

Given the formula for loudness L :

$$L=10\log\left(\frac{I}{I_0}\right)$$

Where: $I = 5\times10^{-6}$ watts per square meter (intensity), $I_0 = 10^{-12}$ watts per square meter (reference intensity)
Substitute the values into the formula:

$$L = 10\log\left(\frac{5\times10^{-6}}{10^{-12}}\right) \Rightarrow L = 10\log\left(5\times10^{6}\right) =$$

Since, $\log\left(10^{6}\right) = 6$ and $\log(5) \approx 0.6990$. Then,

$$10(\log(5) + \log(10^6))$$
$$L = 10\times(0.6990 + 6) = 10\times6.6990 = 66.99$$

Thus, the loudness of the sound is approximately 67 decibels.

(b): If the intensity is doubled, what will be the new loudness in decibels?
When the intensity is doubled, the new intensity I' becomes:

$$I' = 2\times I = 2\times5\times10^{-6} = 1\times10^{-5} \text{ watts per square meter}$$

Substitute this new intensity I' into the loudness formula:

$$L' = 10\log\left(\frac{I'}{I_0}\right) = 10\log\left(\frac{1\times10^{-5}}{10^{-12}}\right) \Rightarrow L' = 10\log\left(1\times10^{7}\right)$$

Since $\log\left(10^{7}\right) = 7$, $L' = 10\times7 = 70$ decibels. Thus, the new loudness is 70 decibels.

2. **The formula for the pH of a solution is $\text{pH} = -\log\left[H^{+}\right]$, where $\left[H^{+}\right]$ is the concentration of hydrogen ions in moles per liter. A solution has a pH of 3.5.**
   a) Calculate the hydrogen ion concentration $\left[\text{H}^{+}\right]$ of the solution.
   b) If another solution has 10 times the hydrogen ion concentration of this solution, what will be its pH ?
   Solution:

(a): Calculating the hydrogen ion concentration $\left[\mathbf{H}^{+}\right]$ of the solution

The formula for pH is given as:

$$\text{pH} = -\log\left[H^{+}\right]$$

We are told that the pH of the solution is 3.5. To find $\left[\text{H}^{+}\right]$, rearrange the formula:

$$\left[H^{+}\right] = 10^{-\text{pH}} \Rightarrow \left[H^{+}\right] = 10^{-3.5}$$

Using a calculator to evaluate $10^{-3.5}$. $\left[H^{+}\right] \approx 3.16\times10^{-4}$ moles per liter

So, the hydrogen ion concentration is approximately 3.16×10^{-4} moles per liter.

(b): If another solution has 10 times the hydrogen ion concentration of this solution, what will be its pH?

If the hydrogen ion concentration of the new solution is 10 times that of the original solution, the new concentration $\left[H^+\right]'$ is $\left[H^+\right]' = 10\times\left[H^+\right] = 10\times 3.16\times 10^{-4} = 3.16\times 10^{-3}$ moles per liter

To find the new pH, use the formula:

$\text{pH}' = -\log\left[H^+\right]' \Rightarrow \text{pH}' = -\log\left(3.16\times 10^{-3}\right)$

Using logarithmic properties, $\text{pH}' \approx -\left(\log 3.16 + \log 10^{-3}\right) = -\left(0.5-3\right) = 2.5$

Thus, the pH of the new solution is 2.5.

3. **A bacterial culture grows exponentially according to the formula $N(t) = N_0 e^{kt}$, where N_0 is the initial population, k is the growth rate constant, and t is time in hours.**
   a) If the initial population is 200 and doul ↓ in 5 hours, find the value of the growth constant k.
   b) Using the value of k, calculate the population after 10 hours.

Solution:

(a): Finding the value of the growth constant k

The bacterial culture grows according to the formula:

$$N(t) = N_0 e^{kt}$$

Where: N_0 is the initial population, k is the growth rate constant, t is time in hours.

Given: $N_0 = 200$ (initial population). The population doubles in 5 hours, so $N(t) = 2N_0 = 400$ when $t = 5$.

Substitute the known values into the equation:

$400 = 200e^{5k} \Rightarrow 2 = e^{5k}$

Take the natural logarithm "In" of both sides and solve for k.

$\ln(2) = 5k \Rightarrow k = \dfrac{\ln(2)}{5} \approx \dfrac{0.693}{5} = 0.1386$ per hour

So, the growth rate constant is approximately $k \approx 0.1386$.

(b): Calculate the population after 10 hours

Using the value of k, we can calculate the population after 10 hours using the formula:

$$N(t) = N_0 e^{kt}$$

Substitute the values: $N_0 = 200$, $k = 0.1386$ and $t = 10$. We have,

$$N(10) = 200e^{0.1386\times 10} = 200e^{1.386}$$

Using a calculator to find $e^{1.386} \approx 4.000$. So, $N(10) = 200\times 4.000 = 800$

Thus, the population after 10 hours is 800.

4. **In the year 2010, the city of Multan had a population of 200,000 people. Due to a government-led reforestation campaign, the number of trees planted in the city has been increasing exponentially at a rate of 5% per year. Initially, there were 50,000 trees. Using the exponential growth formula**

$$P(t)=P_0e^{rt}$$

apply the laws of logarithms to determine the year when the number of trees will reach 100,000.

Solution:

We are given the exponential growth formula:

$$P(t)=P_0e^{rt}$$

Where: P_0 is the initial population of trees, r is the growth rate, t is the time in years, $P(t)$ is the population of trees after t years.

Given that: $P_0=50,000$ trees (initial number of trees), $P(t)=100,000$ trees (final number of trees), $r=0.05$ (growth rate of 5% per year).

We need to find t, the number of years it will take for the number of trees to reach 100,000 .

$100,000=50,000e^{0.05t} \implies 2=e^{0.05t}$

Take the natural logarithm "In" of both sides: $\ln(2)=0.05t \implies t=\dfrac{\ln(2)}{0.05}$

Since, $\ln(2)\approx 0.693$. Therefore, $t=\dfrac{0.693}{0.05}=13.86$

It will take approximately 14 years for the number of trees to reach 100,000. Thus, the year when this will happen is,

$$2010+14=2024$$

So, the trees will reach 100,000 in the year 2024.